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ABOUT AN APPLICATION OF GAMES WITH FIXED ORDER OF MOVEMENTS

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În acest articol este descris în forma unui joc de două persoane cu ordinea fixată a mişcărilor, procesul de gestionarea a relaţiilor dintre întreprinderile agricole şi cele industriale (de prelucrare a producţiei agricole). Pentru jocul formulat este determinat profitul maxim garantat al jucătorilor.

In the relations between the units of production of raw material in agriculture and the industrial units for processing from the processing industry there is a non-coincidence of interests. The processing units are interested in the fact of increasing the period of processing the ran material and in the possibility of an uniform repartition of the ran material during the processing period. If the prices of acquisition are fixed, the producing units have an interest to more fertile varieties, which leads to the decreasing of the processing period and to the appearance of tops, to losses of ran material and to the diminution of nutritive values and technological amounts of the final production. So here appears a problem of regulation of the relations between the processing industry and the agriculture. For that purpose one may introduce some additions to the existent prices of acquisition.

This work is an attempt of the application of the results of work [1] for mathematical modeling the process of the regulation of the relations between the agriculture and the processing industry in the problem of determination of the best proportions of the crops of tomatoes by the regulation of prices of acquisition of tomatoes.

The situation that appears in the relation between the units of production of tomatoes and the processing units of tomatoes can be considered as a game of two players with complete information and non-contradictory purposes. The players (processing industry and agriculture) have a purpose: to mutually maximize the functionals:

$$W_1 = c\sum_{k=1}^{n} z_k - \sum_{k=1}^{n}\sum_{i=1}^{m}(p_{ik}+\Delta p_{ik})y_{ik}x_i - \sum_{k=1}^{n} H_k z_k$$

and

$$W_2 = \sum_{k=1}^{n}\sum_{i=1}^{m}(p_{ik}+\Delta p_{ik})y_{ik}x_k - \sum_{i=1}^{m} c_i \sum_{k=1}^{n} y_{ik}x_i$$

with the following restrictions:

$$\begin{aligned}
&\sum_{i=1}^{m} x_i \le S;\\
&\sum_{i=1}^{m} y_i x_i \ge A;\\
&z_k \le A_K,\ k=1,2,\ldots,n;\\
&\sum_{k=1}^{n}\sum_{i=1}^{m}(\rho_i-\rho)y_{ik}x_i \ge 0;\\
&z_k b_k^r \le B_k^r,\ r=1,2,\ldots,R;\\
&\sum_{i=1}^{m} q_{ik}^l x_i \le Q_k^l,\ l=1,2,\ldots,\widehat{l},\ k=1,2,\ldots,n,\\
&z_k \le \sum_{i=1}^{m}\frac{y_{ik}}{a_i}x_i, k=1,2,\ldots,n.
\end{aligned}$$

where is:
H_k – the expenses necessary for the production of one unit of production in the period k;
x_i – the area tilled with tomatoes of the variety $i,\ i = 1, 2, \ldots, m$;
S – the area which can be cultivated with tomatoes;
p_{ik} – the price of acquisition of tomatoes of i variety in the period $k,\ k = 1, 2, \ldots, n$;
y_{ik} – the crop of tomatoes of i variety in the period k;
a_i – the amount of tomatoes of i variety necessary for the production of one unit of final production;
$y_i = \sum\limits_{k=1}^{n} y_{ik}$ – the crop of tomatoes of i variety;
c_i – the expenses supported for the cultivation of i variety on a unit of surface;
Δp_{ik} – the addition to the price of acquisition of tomatoes of i variety in the period k;
A – the amount of tomatoes necessary to fulfill the plan of production;
A_k – the maximum amount of production in the period k;
c – the selling price of production;
ρ_i – the percentage of dry substances contained in the tomatoes of i variety;
ρ – the necessary percentage of dry substances;
z_k – the amount of production in the period k;
W_1 – the profit of the processing industry;
W_2 – the profit of the agricultural unit;
b_k^r – the expenses of resources of r type for the production of one unit of production in the period $k,\ r = 1, 2, \ldots, R$;
B_k^r – the amount of resources of r type available in the processing industry in the period k;
q_{ik}^l – the expenses of resources of type l, necessary to cultivate a unit of tomatoes of i variety in the period $k,\ l = 1, 2, \ldots, \widehat{l}$;
Q_k^l – the amount of resources of l type available in the agricultural unit in the period k;
$x = (x_1, x_2, \ldots, x_m)$ – the vector of the decisions of the agricultural unit;
$z = (z_1, z_2, \ldots, z_n)$ and $\Delta p = (\Delta p_{ik})$ – the vectors of the decisions of the processing industry.

Let's suppose, that the processing industry has complete information about the choice of the decision of the agricultural unit. In this case one may formulate a game of the type Γ_2 [2], where the first player, which is the processing industry, chooses its strategy Δp as a function of x. If the first has chosen such a strategy, its profit depends only on z, and, so, the processing industry can determine, for the given x, that z, which will maximize its profit. On this purpose it is necessary to solve the following problem of linear programming:

let's determine the maximum W_1 with the restrictions

$$z \in Z_k = \begin{cases} z_k \le \sum\limits_{i=1}^{m} \dfrac{y_{ik}}{a_i}, k = 1, 2, \ldots n, \\ z_k \le A_k, k = 1, 2, \ldots n, \\ z_k b_k^r \le B_k^r, r = 1, 2, \ldots R, k = 1, 2, \ldots n. \end{cases}$$

Let

$$\overline{W_1}(x) = \max_{z \in Z_k} \left[c \sum_{k=1}^{n} z_k - \sum_{k=1}^{n} H_k z_k \right].$$

Then the profit of the processing industry will be

$$W_1(x, \Delta p) = \overline{W_1}(x) - \sum_{k=1}^{n} \sum_{i=1}^{m} (p_{ik} + \Delta p_{ik}) y_{ik} x_i.$$

As a result one may obtain the following game. Two players, the first being the processing industry and the second being the agricultural unit, have the purpose to maximize mutually the functionals:

$$W_1(x,\Delta p)=\overline{W_1}(x)-\sum_{k=1}^{n}\sum_{i=1}^{m}(p_{ik}+\Delta p_{ik})y_{ik}x_i$$

and

$$W_2(X,\Delta p)=\sum_{k=1}^{n}\sum_{i=1}^{m}(p_{ik}+\Delta p_{ik})y_{ik}x_i-\sum_{i=1}^{m}\sum_{k=1}^{n}c_i y_{ik}x_i,$$

where x is the vector of the decisions of the second player, and Δp is the vector of the decisions of the first player.

The admissible decisions of the players are mutually given by the multitudes:

$$x\in X=\begin{cases}\sum\limits_{i=1}^{m}x_i\le S,\\ \sum\limits_{k=1}^{n}\sum\limits_{i=1}^{m}(\rho_i-\rho)\,y_{ik}x_i\ge 0,\\ \sum\limits_{i=1}^{m}q_{ik}^{l}x_i\le Q_k^{l},\ \ l=1,2,\ldots,\widehat{l},\ \ k=1,2,\ldots,n\end{cases}$$

and

$$\Delta p\in\Delta P=\{\Delta p_{ik}\ge 0,\ i=1,2,\ldots,m,\ k=1,2,\ldots,n\}.$$

In the formulated game one may apply the Ghermeyer's theorem [2].

Let's introduce the notations:

$$L=\max_{x\in X}\min_{\Delta p\in\Delta P}W_2(x,\Delta p),$$
$$E_2=\left\{x\in X:\ \min_{\Delta p\in\Delta P}W_2(x,\Delta p)=L\right\},$$
$$D=\{(x,\Delta p):\ W_2(x,\Delta p)>L,\ x\in X,\ \Delta p\in\Delta P\},$$
$$M=\min_{x\in E_2}\max_{\Delta p\in\Delta P}W_1(x,\Delta p)=\min_{x\in E_2}\left(\overline{W_1}(x)-\sum_{k=1}^{n}\sum_{i=1}^{m}p_{ik}y_{ik}x_i\right),$$
$$K=\begin{cases}\sup\limits_{D}W_1(x,\Delta p), & if D\ne\emptyset,\\ -\infty, & if D=\emptyset.\end{cases}$$

Because in the examined game $D\ne\emptyset$, $K=\sup\limits_{D}W_1(x,\Delta p)$. Besides hat M represents the value of the function $W_1(x,\Delta p)$ in the point $(\overline{x},0)$ where $\overline{x}$ is on indefinite point from the multitude E_2. And, as E_2 belongs to the border of the multitude D, $K\ge M$. So according to Ghermeyer's theorem, the guaranteed profit of the processing industry is equal to k, otherwise our problem is reduced to the determination of the vectors Δp and x, which realizes the extreme of the function $W_1(x,\Delta p)$ on the multitude D.

References

[1] I. B. Ghermeyer, *Ob igrah n lits s ficsirovannoi posledovatelinostiu hodov*, DAN SSSR, 198, N. 5, 1971, p. 1001–1004 (in Russian).

[2] I. A. Vatel, F. I. Eresco, *Matematica conflicta i sotrudnichestva*, Moskva, "Znanie", 1973. (in Russian).

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