DECOMPOSITION OF OPTIMUM DESIGN PROBLEMS

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În lucrare sunt prezentate rezultalele rezolvării problemei de optimizare a forţei critice de pierdere a stabilităţii placii ortotropice prin alegerea raţională a unghiului de ortotropie $\varphi(x,y)$. Se cercetează placa ortotropică din sticlă-plastic rezemată rigid solicitată cu forţele externe de compresiune la contur. Se presupune că direcţia forţelor aplicate nu se schimbă în procesul deformării. Din cauza simetriei, problema a fost rezolvată prin metoda de decompoziţie.

Introduction

Among a variety of systems the symmetrical ones occupy an important place. The symmetry of a system greatly influences on the system's properties. For example, the symmetry of the mechanical system defines the specificity of structure of free oscillations frequency spectrum, buckling loads, formation of optimal solutions. Symmetry means an intrinsic property of a mathematical object which causes it to remain invariant under certain classes of transformations (such as rotation, reflection, inversion, or more abstract operations) Thus, the square is invariant to rotations on angles divisible by 90 and operations of reflection along the straight lines which join the middle points of opposite sides, and along diagonals of square. The mathematical study of symmetry is systematized and formalized in d group theory.

The analysis of properties of the symmetrical problem of optimization allows to make its decomposition, i.e. replace the original problem with the series of subtasks (classes), defined on some subregion Ω_0 (called an elementary cell) of definition range of the initial problem. That is especially important in numerical calculations of the problem of optimization as it allows to reduce considerably the dimensions of arrays obtained at the stage of discretization and to decrease the computational resources. Also we can obtain optimal solution properties at the phase of analysis before solving the problem of optimization.

We shall note, that the elementary cell Ω_0 contains new parts of boundary, on fig.1 denote through Γ_d^0 ,Γ_y^0 Obtaining boundary conditions on new parts of domain boundaries Ω_0 therefore, is obviously important. Group theory and representation theory was applied for the analysis of the symmetric structures and obtaining boundary conditions.

In the suggested article the behavior of the elastic orthotropic plate loaded with compressing forces, applied to exterior contour of the plate is investigated. The direction of the exterior loadings does not vary during deformation. Anisotropic properties of the plate are described by function $\varphi(x,y)$ - an angle between axes of orthotropy in the point with coordinates (x,y) and axes of fixed co-ordinates connected with the plate. Applied loadings vary proportionally to parameter λ and at some value of this parameter called critical, a flat plate loses its stability, buckles and accepts the curved form described by function $w(x,y)$ $w(x,y)$ - displacement from plane). Critical buckling loads of an anisotropic plate, alongside with other mechanical and geometrical characteristics, considerably depend on the distribution of anisotropy angle. The design problem of the orthotropic plate with extreme value of critical buckling load with respect to anisotropy angle $\varphi(x,y)$ is examined.

Mathematical formulation of the problem of optimization

Critical buckling load can be determined from condition of functional minimum [1]:

$$\lambda = \frac{\int\limits_{\Omega} V(\varphi;w)dxdy}{-\int\limits_{\Omega} A(\varphi;w)dxdy} \to \min_{w}, \tag{1}$$

where $V(\varphi,w)$ - denseness of potential energy of bend deformation, $A(\varphi,w)$ - denseness of work of external forces, $w(x,y)$ - the bend of the plate at its buckling, Ω - area occupied by the plate.

The denseness of potential energy of lateral bending of orthotropic plate and denseness of work of applied forces at lateral deformation are determined as:

$$V(\varphi,w)=\tfrac{1}{2}[D_{11}(\varphi)w_{xx}^2+2D_{12}(\varphi)w_{xx}w_{yy}+D_{22}(\varphi)w_{yy}^2+ \\ +4(D_{16}(\varphi)w_{xx}+D_{26}(\varphi)w_{yy})w_{xy}+4D_{66}(\varphi)w_{xy}^2], \tag{2}$$

$$A(\varphi,w)=\frac{1}{2}(N_{xx}^0(\varphi)w_x^2+2N_{xy}^0(\varphi)w_xw_y+N_{yy}^0(\varphi)w_y^2), \tag{3}$$

where $N_{xx}^0=h\sigma_{xx}^0(\varphi),\ N_{xy}^0=h\sigma_{xy}^0(\varphi),\ \ N_{yy}^0=h\sigma_{yy}^0(\varphi)$, $h-$ width of the plate, $\sigma_{xx}^0(\varphi),\ \sigma_{xy}^0(\varphi),\ \sigma_{yy}^0(\varphi)$ – components of stress tensor of the plate caused by exterior loads, corresponded to value $\lambda=1$, $D_{ij}(\varphi)$ – bending rigidities of the plate. Components of stress tensor satisfy to equilibrium equations:

$$\frac{\partial\sigma_{xx}^0(\varphi)}{\partial x}+\frac{\partial\sigma_{xy}^0(\varphi)}{\partial y}=0,\qquad \frac{\partial\sigma_{xy}^0(\varphi)}{\partial x}+\frac{\partial\sigma_{yy}^0(\varphi)}{\partial y}=0 \tag{4}$$

and to the generalized Hooke's law [2]

$$\begin{aligned}
\sigma_{xx}^0(\varphi)&=A_{11}(\varphi)\frac{\partial u}{\partial x}+A_{12}(\varphi)\frac{\partial v}{\partial y}+A_{16}(\varphi)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\\
\sigma_{yy}^0(\varphi)&=A_{12}(\varphi)\frac{\partial u}{\partial x}+A_{22}(\varphi)\frac{\partial v}{\partial y}+A_{26}(\varphi)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right),\\
\sigma_{xy}^0(\varphi)&=A_{16}(\varphi)\frac{\partial u}{\partial x}+A_{26}(\varphi)\frac{\partial v}{\partial y}+A_{66}(\varphi)\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)
\end{aligned} \tag{5}$$

Bending rigidities $D_{ij}(\varphi)$ are defined by expressions [3]:

$$\begin{aligned}
D_{11}(\varphi)&=D_x\cos^4\varphi+2D_3\cos^2\varphi\sin^2\varphi+D_y\sin^4\varphi\\
D_{12}(\varphi)&=D_1+(D_x+D_y-2D_3)\cos^2\varphi\sin^2\varphi\\
D_{22}(\varphi)&=D_x\sin^4\varphi+2D_3\sin^2\varphi\cos^2\varphi+D_y\cos^4\varphi\\
D_{16}(\varphi)&=(D_x\cos^2\varphi-D_y\sin^2\varphi-D_3\cos 2\varphi)\cos\varphi\sin\varphi,\\
D_{26}(\varphi)&=(D_x\sin^2\varphi-D_y\cos^2\varphi+D_3\cos 2\varphi)\cos\varphi\sin\varphi\\
D_{66}(\varphi)&=D_{xy}+(D_x+D_y-2D_3)\cos^2\varphi\sin^2\varphi
\end{aligned} \tag{6}$$

where $D_1=\dfrac{E_1\nu_{12}h^3}{12(1-\nu_{12}\nu_{21})}$, $D_x=\dfrac{E_1h^3}{12(1-\nu_{12}\nu_{21})}$, $D_y=\dfrac{E_1h^3}{12(1-\nu_{12}\nu_{21})}$, $D_{xy}=\dfrac{G_1h^3}{12}$, $D_3=D_1+2D_{xy}$

Expressions for coefficients of orthotropy $A_{ij}(\varphi)$ in the fixed co-ordinates (x,y) are analogous to formulas for coefficients $D_{ij}(\varphi)$ with correspondence $D_{ij}(\varphi)\to A_{ij}(\varphi)$

$$D_x\to A_{11}^0=\frac{E_1}{(1-\nu_{12}\nu_{21})},\ D_y\to A_{22}^0=\frac{E_2}{(1-\nu_{12}\nu_{21})},\ D_{xy}\to A_{66}^0=G_{12},\ D_1\to A_{12}^0=\frac{E_1\nu_{12}}{(1-\nu_{12}\nu_{21})},$$

where $E_1,E_2,G_{12},\nu_{12},\nu_{21}$ $(E_1\nu_{12}=E_2\nu_{21})$ - are the constants of the anisotropic material. The considered problem can be formulated in the following way: To determine the distribution of anisotropy angle for which critical buckling load accepts a extreme

$$\varphi(x,y):\quad \lambda(\varphi(x,y))\to\max_{\varphi(x,y)} \tag{7}$$

Discretization

The finite element method was applied for numerical solution of the problem. The plate area was divided on triangular elements. The function of bend $w(x,y)$ and the function of displacements in plane $(u(x,y),v(x,y)$ are approximated with polynomials [4].

The finite element formulation of the problem of determination of critical buckling loads is following

$$\lambda(\vec{\varphi}) = \min_{\{\vec{w}\}} \frac{(\{\vec{w}\},[K_{изг}(\vec{\varphi})\{\vec{w}\}])}{-(\{\vec{w}\},[K_g(\vec{\varphi},\{\vec{q}\})]\{\vec{w}\})}, \tag{8}$$

where $\{\vec{w}\}, \{\vec{q}\}$ - global vectors of the nodal bending displacements and plain displacements respectively , $\bar{\varphi} = (\varphi_1,....,\varphi_{N_e})$ - the vector of angles of anisotropy. We consider that anisotropy angle is constant for each element.

Through $[K_{изг}(\vec{\varphi})\{\vec{w}\}],\ [K_g(\vec{\varphi},\{\vec{q}\})]$ the global stiffness matrix and a global geometrical stiffness matrix of the plate correspondingly are denote. The global vector is obtained as solution of plane problem of theory of elasticity

$$[K_{пл}(\vec{\varphi})]\{\vec{q}\} = \{\vec{f}\}, \tag{9}$$

where $[K_{пл}(\vec{\varphi})]$ })] - global stiffness matrix of plane problem, $\{\vec{f}\}$- global vector of nodal loads in plane that correspond to value $\lambda = 1$.

The finite-element formulation of the problem of optimization is defined as follows: to find anisotropy angles $\vec{\varphi}^{опт} = (\varphi_1,\varphi_2,....,\varphi_{N_e})$ that give an extreme value to the functional

$$\lambda(\vec{\varphi}) \to \max_{\varphi_1,\varphi_2,....\varphi_{N_e}} \quad (\min_{\varphi_1,\varphi_2,....\varphi_{N_e}}), \tag{10}$$

where vector $\{\vec{w}\}$ satisfies the algebraic eigenvalue problem

$$[K_{изг}(\vec{\varphi})]\{\vec{w}\} = -\lambda[K_g(\vec{\varphi},\{\vec{q}\}]\{\vec{w}\}, \tag{11}$$

and global plane displacement vector satisfies the equations

$$[K_{пл}(\vec{\varphi})\{\vec{q}\} = \{\vec{f}\} \tag{12}$$

Due to the symmetry of area, loadings, boundary conditions optimum solution is searched in the class of the symmetrical functions $\varphi(x,y)$. By virtue of symmetry, the problem of determination of critical buckling load can be divided into 5 classes. Each of the problem is solved on elementary cell. Denote through $\theta = \frac{\partial w}{\partial y},\ \psi = -\frac{\partial w}{\partial x}$

Boundary conditions for these classes are:

1 class: $\psi(x,y) = 0$ on Γ_y^0 $\quad \psi(x,y) = -\theta(x,y)$ on Γ_d^0

2 class: $w(x,y) = 0,\ \theta(x,y) = 0$ on Γ_y^0 $\quad w(x,y) = 0,\ \psi(x,y) = \theta(x,y)$ on Γ_d^0

3 class: $w_2(x,y) = 0,\ \theta_1(x,y) = 0,\ \psi_2(x,y) = 0$ on Γ_y^0

$w_2(x,y) = w(x,y)_1,\ \theta_1(x,y) = -\psi_2(x,y),\ \psi_1(x,y) = \theta_2(x,y)$ on Γ_d^0

4 class: $\psi(x,y) = 0$ on Γ_y^0 $\quad w(x,y) = 0,\ \psi(x,y) = \theta(x,y)$ on Γ_d^0

5 class: $w(x,y) = 0,\ \theta(x,y) = 0$ на Γ_y^0 $\quad \psi(x,y) = -\theta(x,y)$ на Γ_d^0

Numerical solution of the problem of optimization

Numerical calculations have been performed for the square plate made of fiberglass plastic, rigidly clamped boundary, characterized by the following values of elastic modules: $E_1/E_2 = 3$, $E_1/G_{12} = 6$,

$E_1 / G_{12} = 6$, $\nu_{12} = 0.83$, $\nu_{21} = 0.25$, width and thickness of the plate were considered equal to 1. The plate was divided into $N = 800$ finite elements. The division scheme is shown on Fig. 1.

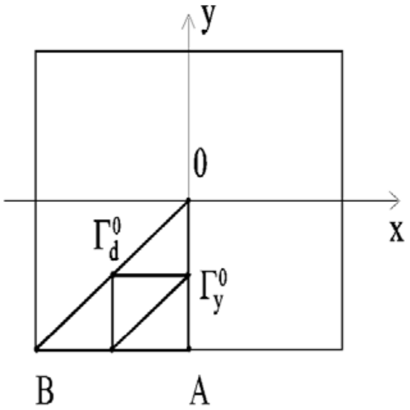

Fig.1. The division scheme on triangular elements of square plate.

The optimum problem was solved by the sequential optimization method. The new approximation $\vec{\varphi}_{n+1}$ according to this method was obtained by the formula

$$\vec{\varphi}_{n+1} = \vec{\varphi}_n + \tau\Lambda(\vec{\varphi}_n), \tag{13}$$

where τ - a step in the direction of gradient, $\Lambda(\vec{\varphi}_n)$-the value of gradient at the previous approximation. The necessary condition of the optimality is: $\Lambda = 0$ As the initial approximation of anisotropy angle was selected on the 1/8 part of plate. For selected initial approximation critical buckling load was equal to $\lambda_1 = 2.811$, the norm of gradient, calculated as $\sqrt{\sum_i \Lambda_i^2}$ was 0.002. The obtained optimum solution is shown on fig. 2. The tangent line in each point of the lines coincides with the direction of the greater orthotropy axes. For the optimum solution the norm of the gradient is equal $0.5 * 10^{-4}$, and value of critical buckling $\lambda_1^{opt} = 3.854$

For comparison we shall show respect of the optimal value of critical buckling load to the value of critical buckling load for $\varphi(x, y) \equiv 0$ $\dfrac{\lambda_1(\varphi^{opt})}{\lambda_1(\varphi \equiv 0)} = 1.52$

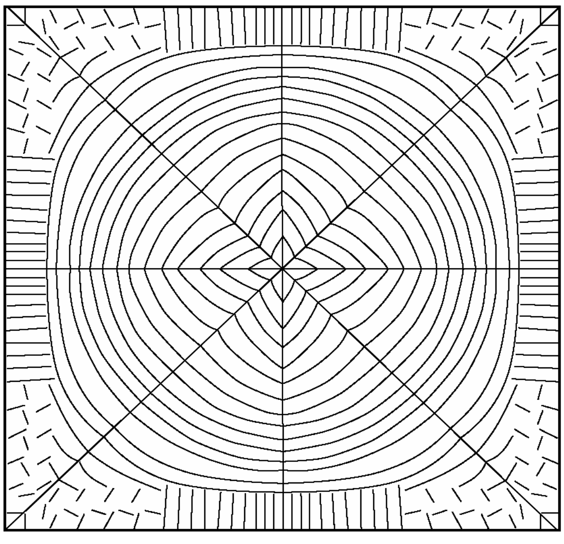

Fig.2. Optimum distribution of anisotropy angle for the rigidly clamped boundary plate.

Thus, the distribution of the anisotropy angle essentially affects the critical buckling load.

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