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ON QUASIGROUPS WITH SOME MINIMAL IDENTITIES

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Quasigroups with two identities (of types T_1 and T_2) from Belousov-Bennett classification are considered. It is proved that a π-quasigroup of type T_2 is also of type T_1 if and only if it satisfies the identity $yx \cdot x = y$ (the "right keys law"), so π-quasigroups that are of both types T_1 and T_2 are RIP-quasigroups. Also, it is proved that π-quasigroups of type T_2 are isotopic to idempotent quasigroups. Necessary and sufficient conditions when a π-quasigroup of type T_2 is isotopic to a group (an abelian group) are found. It is shown that the set of all π-quasigroups of type T_2 isotopic to abelian groups is a subvariety in the variety of all π-quasigroups of type T_2 and that π-T-quasigroups of type T_2 are medial quasigroups. Using the symmetric group on $Q \times Q$, some considerations for the spectrum of finite π-quasigroups $(Q,\cdot)$ of type T_1 are discussed.
Keywords: *minimal identities, π-quasigroup, group's isotopes, spectrum.*

ASUPRA CVASIGRUPURILOR CU UNELE IDENTITĂȚI MINIMALE

Sunt considerate cvasigrupuri cu două identități (de tipurile T_1 şi T_2) din clasificarea Belousov-Bennett. Se demonstrează că un π-cvasigrup de tipul T_2 este un π-cvasigrup de tipul T_1 dacă şi numai dacă el verifică identitatea $yx \cdot x = y$ (legea „cheilor la dreapta"), deci π-cvasigrupurile care sunt simultan de tipul T_1 şi T_2 sunt RIP-cvasigrupuri. De asemenea, se arată că π-cvasigrupurile de tipul T_2 sunt izotope unor cvasigrupuri idempotente. Sunt determinate condițiile necesare şi suficiente ca un π-cvasigrup de tipul T_2 să fie izotop unui grup (grup abelian). Astfel, se obține că π-cvasigrupurile de tipul T_2 izotope unor grupuri abeliene formează o subvarietate în varietatea tuturor π-cvasigrupurilor de tipul T_2 şi că π-T-cvasigrupurile de tipul T_2 sunt mediale. Este dată o caracterizare a spectrului π-cvasigrupurilor finite $(Q,\cdot)$ de tipul T_1 în limbajul substituțiilor mulțimii $Q \times Q$.
Cuvinte-cheie: *identități minimale, π-cvasigrupuri, izotopi ai grupurilor, spectru.*

A binary quasigroup (Q, A) is called a π-quasigroup of type $[\alpha, \beta, \gamma]$, where $\alpha, \beta, \gamma \in S_3$, if it satisfies the identity

$$^{\alpha}A(x, {}^{\beta}A(x, {}^{\gamma}A(x,y))) = y, \qquad (1)$$

where ${}^{\sigma}A$ denotes the σ-parastrophe of A. A classification of the identities (1) was given by V. Belousov [2] and, independently, by F. Bennett [4]. The corresponding types of the identities from this classification are: $T_1 = [s, s, s]$, $T_2 = [s, s, l]$, $T_4 = [s, s, lr]$, $T_6 = [s, l, lr]$, $T_{10} = [s, lr, l]$, $T_8 = [s, rl, lr]$, $T_{11} = [s, lr, rl]$, where $l = (13), r = (23)$.

Recall that a quasigroup $(Q,\cdot)$ is called: a π-quasigroup of type T_2 if it satisfies the identity:

$$x \cdot (y \cdot yx) = y, \qquad (2)$$

and is a π-quasigroup of type T_1 if it satisfies the identity:

$$x \cdot (x \cdot xy) = y. \qquad (3)$$

π-Quasigroups of type T_1 are studied in [2,4,8,9]. The spectrum of the identity (3) was considered in [4]: it is precisely $q \equiv 0$ or $1 \ (mod\ 3)$, except for $q = 6$. Necessary conditions when a finite π-quasigroup of type T_1 has the order $q \equiv 0 \ (mod\ 3)$ are given in [9]. In particular, it is proved in [9] that a π-quasigroup of type T_1 for which the order of inner mappings group is not divisible by three always has a left unit. Necessary and sufficient conditions when the identity (3) is invariant under the isotopy of quasigroups (loops) and π-quasigroups of type T_1 isotopic to groups, in particular π-T-quasigroups of type T_1, are considered in [9]. The holomorph of π-quasigroups of type T_1 was studied in [6].

π-Quasigroups of both types T_1 and T_2 are considered in the present work. It is proved that a π-quasigroup of type T_2 has also the type T_1 if and only if it satisfies the right keys law, so π-quasigroups that are of both types T_1 and T_2 are RIP-quasigroups. Also, it is proved that π-quasigroups of type T_2 are isotopic to idempotent quasigroups. Necessary and sufficient conditions when a π-quasigroup of type T_2 is isotopic to a group (an abelian group) are found. It is shown that the set of all π-quasigroups of type T_2 isotopic to abelian groups is a subvariety in the variety of all π-quasigroups of type T_2 and that π-T-quasigroups of type T_2 are medial quasigroups. Also, some considerations for the spectrum of finite π-quasigroups of type T_1 are discussed in present work.

Proposition 1. *A π-quasigroup $(Q,\cdot)$ of type T_2 is a π-quasigroup of type T_1 if and only if $(Q,\cdot)$ satisfies the identity*

$$yx \cdot x = y. \tag{4}$$

Proof. If $(Q,\cdot)$ is a π-quasigroup of types T_2 and T_1 then, replacing x by yx in (2), we get:

$$y = yx \cdot \big(y \cdot (y \cdot yx)\big) = yx \cdot x,$$

$\forall x, y \in Q$. Conversely, if $(Q,\cdot)$ is a π-quasigroup of type T_2 and satisfies the identity $yx \cdot x = y$ then, replacing x by yx in (2) we have: $yx \cdot \big(y \cdot (y \cdot yx)\big) = y = yx \cdot x \Rightarrow y \cdot (y \cdot yx) = x$, $\forall x, y \in Q$, i.e. $(Q,\cdot)$ is a π-quasigroup of type T_1. □

Corollary. *π-Quasigroup having both types T_2 and T_1 are RIP-quasigroups.*

Example 1. The quasigroup $(Q,\cdot)$, where $Q = \{1,2,3,4\}$, given by its left translations $L_1 = (234), L_2 = (124), L_3 = (132), L_4 = (143)$ is a π-quasigroup of both types T_1 and T_2.

Remark 1. π-Loops of type T_2 are trivial. Indeed, if $(Q,\cdot)$ is a π-loop of type T_2 with the unit e then, taking $x = e$ in (2) we get $y \cdot y = y$, so $y = e$, i.e. $|Q| = 1$.

Remark 2. π-Quasigroups of both types T_1 and T_2 are anticommutative. Indeed, if $(Q,\cdot)$ is a π-quasigroup of types T_2 and T_1 and $x \cdot y = y \cdot x$, then $x \cdot (y \cdot yx) = y$ and $x \cdot (x \cdot yx) = y$, so $x \cdot (x \cdot yx) = x \cdot (y \cdot yx)$, which implies $x \cdot yx = y \cdot yx$, so $x = y$.

Proposition 2. *A π-quasigroup $(Q,\cdot)$ of type T_2 is isotopic to an abelian group if and only if it satisfies the identity:*

$$[y \cdot (v \cdot vu)] \cdot [(y \cdot (v \cdot vu)) \cdot x] = [y \cdot (v \cdot vx)] \cdot [(y \cdot (v \cdot vx)) \cdot u]. \tag{5}$$

Proof. It is shown in [1] that a quasigroup $(Q,\cdot)$ is isotopic to an abelian group if and only if it satisfies the identity

$$x \backslash (y \cdot (u \backslash v)) = u \backslash (y \cdot (x \backslash v)), \tag{6}$$

where "\" is the right division in $(Q,\cdot)$. If $(Q,\cdot)$ is a π-quasigroup of type T_2 then from (2) follows:

$$x \backslash y = y \cdot yx, \tag{7}$$

$\forall x, y \in Q$. Using (7) in (6), we get the identity (5). □

Corollary 1. *π-Quasigroups of type T_2 isotopic to abelian groups are π-quasigroups of type T_2.*

Proof. Let $(Q,\cdot)$ be a π-quasigroup of type T_1 isotopic to an abelian group. Taking $u = v = y$ in (5) and using (3), we get $y \cdot yx = x \cdot xy$, which implies $x = y \cdot (y \cdot yx) = y \cdot (x \cdot xy), \forall x, y \in Q$, i.e $(Q,\cdot)$ is a π-quasigroup of type T_2. □

Corollary 2. *The set of all π-quasigroups of type T_2 isotopic to abelian groups is a subvariety in the variety of all π-quasigroups of type T_2.*

Example 2. The quasigroup $(Q,\cdot)$, where $Q = \{1, 2, 3, 4\}$, given by its left translations $L_1 = (123), L_2 = (243), L_3 = (142), L_4 = (134)$ is a π-quasigroup of both types T_1 and T_2 and satisfies (5), so $(Q,\cdot)$ is isotopic to an abelian group.

Remark 3. π-Quasigroups of type T_1, isotopic to abelian groups are not always π-quasigroups of type T_2 as shows the following example.

Example 3. The quasigroup $(Q,\cdot)$, where $Q = \{1, 2, 3, 4\}$, given by its left translations: $L_1 = (123), L_2 = (243), L_3 = (134), L_4 = (142)$ is a π-quasigroup of type T_1 and satisfies the identity $x\backslash(y \cdot (u\backslash v)) = u\backslash(y \cdot (x\backslash v))$, so $(Q,\cdot)$ is isotopic to an abelian group. It is easy to verify that $(Q,\cdot)$ is not a π-quasigroup of type T_2.

Proposition 3. *Let $(Q,\cdot)$ be a π-quasigroup of both types T_1 and T_2. $(Q,\cdot)$ is isotopic to a group if and only if it satisfies the identity*

$$x(y \cdot y(zu \cdot v)) = (x(y \cdot yz) \cdot u)v. \tag{8}$$

Proof. It is known [5] that a quasigroup $(Q,\cdot)$ is isotopic to a group if and only if it satisfies the identity

$$x(y\backslash((z/u)v)) = ((x(y\backslash z))/u)v, \tag{9}$$

where "\" and "/" are the right and the left division in $(Q,\cdot)$. If $(Q,\cdot)$ is a π-quasigroup $(Q,\cdot)$ of types T_1 and T_2 then from (4) follows:

$x/y = x \cdot y,$

$\forall x, y \in Q$. Using the last equality in (8), we obtain

$$x(y\backslash((z \cdot u)v)) = ((x(y\backslash z)) \cdot u)v. \tag{10}$$

$(Q,\cdot)$ is a π-quasigroup of type T_1, so $x\backslash y = x \cdot xy$. Using the last equality in (10), we get (8). □

Corollary. *If $(Q,\cdot)$ is a π-quasigroup of types T_1 and T_2, isotopic to a group, then it satisfies the identity*

$$(yz \cdot v)u = (zu \cdot v)(yz \cdot y). \tag{11}$$

Proof. Let $(Q,\cdot)$ be a π-quasigroup of both types T_1 and T_2, isotopic to a group. Taking $x = zu \cdot v$ and using (2) in (8), we get $y = [(zu \cdot v)(y \cdot yz)]u \cdot v$. Multiplying by v from the right, then by u from the right the last equality and using (4), we get $yv \cdot u = (zu \cdot v)(y \cdot yz)$. Replacing y by yz in the last equality and using (4), we obtain (11). □

Example 4. The quasigroup $(Q,\cdot)$, where $Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, given by its left translations:

$$L_1 = (123)(567)(89),\ L_2 = (489)(576),\ L_3 = (132)(498),\ L_4 = (154)(279)(368),$$
$$L_5 = (147)(296)(385),\ L_6 = (195)(287)(346),\ L_7 = (186)(245)(697),$$
$$L_8 = (178)(264)(359),\ L_9 = (169)(258)(374)$$

is a π-quasigroup of both types T_1 and T_2 and satisfies (8), so $(Q,\cdot)$ is isotopic to a group. $(Q,\cdot)$ does not satisfy (5), i. e. is not isotopic to an abelian grup.

Proposition 4. *If a π-quasigroup $(Q,\cdot)$ of types T_1 and T_2 is principal isotopic to an abelian group $(Q,+)$ then $(\cdot) = (+)^{(I, L_0^{(\cdot)}, \varepsilon)}$, where 0 is the neutral element of the group $(Q,+)$ and I is the inversion in $(Q,+)$ $(I: Q \to Q,\ I(x) = -x, \forall x \in Q)$.*

Proof. Let $(Q,\cdot)$ be a π-quasigroup of types T_1 and T_2, principal isotopic to an abelian group $(Q,+)$, so $(+) = (\cdot)^{(R_a^{(\cdot)-1}, L_b^{(\cdot)-1}, \varepsilon)}$ or $x + y = R_a^{(\cdot)-1}(x) \cdot L_b^{(\cdot)-1}(y)$. Taking $x = 0$ in the last equality, we have $y = R_a^{(\cdot)-1}(0) \cdot L_b^{(\cdot)-1}(y)$. Denoting $R_a^{(\cdot)-1}(0)$ by c, the last equality takes the form $y = c \cdot L_b^{(\cdot)-1}(y) = L_c^{(\cdot)} L_b^{(\cdot)-1}(y)$, so $L_c^{(\cdot)} L_b^{(\cdot)-1} = \varepsilon$ or

$$b = c = R_a^{(\cdot)-1}(0). \tag{12}$$

The isotopy $(+) = (\cdot)^{(R_a^{(\cdot)-1}, L_b^{(\cdot)-1}, \varepsilon)}$ implies

$$x \cdot y = R_a^{(\cdot)}(x) + L_b^{(\cdot)}(y) = xa + by. \tag{13}$$

So as $(Q,\cdot)$ is a π-quasigroup of types T_1 and T_2, from Proposition 1 follows that $(Q,\cdot)$ satisfies the identity $yx \cdot x = y$. From (13) and (4) we obtain $R_a^{(\cdot)}\left(R_a^{(\cdot)}(y) + L_b^{(\cdot)}(x)\right) + L_b^{(\cdot)}(x) = y$ or $R_a^{(\cdot)}\left(R_a^{(\cdot)}(y) + x\right) + x = y$, so $R_a^{(\cdot)}\left(R_a^{(\cdot)}(y) + x\right) = y - x$. Taking $x = y$ in the last equality, we have $R_a^{(\cdot)}\left(R_a^{(\cdot)}(x) + x\right) = 0 \Rightarrow R_a^{(\cdot)}(x) + x = R_a^{(\cdot)-1}(0)$. Using (12), we obtain

$$R_a^{(\cdot)}(x) = b - x. \tag{14}$$

Using (14) in (13), we get:

$$x \cdot y = b - x + by. \tag{15}$$

Taking $x = 0$ in (15), we obtain

$$by = -b + 0 \cdot y. \tag{16}$$

From (15) and (16), we have $b - x - b + 0 \cdot y = x \cdot y$ or $x \cdot y = -x + 0 \cdot y$. From the last equality we obtain $(\cdot) = (+)^{(I, L_0^{(\cdot)}, \varepsilon)}$. □

Proposition 5. *Every π-quasigroup of type T_2 is isotopic to an idempotent quasigroup.*

Proof. Let $(Q,\cdot)$ be a π-quasigroup of type T_2 and let $a \in Q$. Then its isotope $(Q,\circ)$, given by the isotopy $T = \left(\varepsilon, R_a^{(\cdot)}, L_a^{(\cdot)-1}\right)$, where $R_a^{(\cdot)}(x) = x \cdot a$ and $L_a^{(\cdot)}(x) = a \cdot x, \forall x \in Q$, is idempotent: $x \circ x = L_a^{(\cdot)}\left(x \cdot R_a^{(\cdot)}(x)\right) = a \cdot (x \cdot xa) = x, \forall x \in Q$. □

Example 5. The quasigroup $(\mathbb{Z}_5,\cdot)$, where $x \cdot y = 3x + 3y \pmod 5, \forall x, y \in \mathbb{Z}_5$, is an idempotent π-quasigroup of type T_2.

Remark 4. π-Quasigroups of type T_2 are admissible so as $x \cdot (y \cdot yx) = y \Rightarrow x \backslash y = y \cdot yx \Rightarrow L_x^{(\backslash)}(y) = y \cdot R_x^{(\cdot)}(y), \forall x, y \in Q$, where $L_x^{(\backslash)}$ is the left translation with x in $(Q,\backslash)$, so $L_x^{(\backslash)}$ is a complete mapping of $(Q,\cdot)$. It is known ([1]) that admissible quasigroups are isotopic to idempotent quasigroups.

Recall that a quasigroup $(Q,\cdot)$ is called a T-quasigroup if there exists an abelian group $(Q,+)$, its automorphisms $\varphi,\psi \in Aut(Q,+)$ and an element $g \in Q$ such that, for every $x,y \in Q$, the following equality holds:

$$x \cdot y = \varphi(x) + \psi(y) + g.$$

The tuple $((Q,+),\varphi,\psi,g)$ is called a T-form and the group $(Q,+)$ is called a T-group of the T-quasigroup $(Q,\cdot)$.

Proposition 6. *A T-quasigroup $(Q,\cdot)$ with the T-form $((Q,+),\varphi,\psi,g)$ is a π-quasigroup of type T_2 if and only if the following conditions hold: 1) $\psi^2(g)+\psi(g)+g=0$; 2) $\varphi = I\psi^3$; 3) $\psi^6+\psi^4=I$, where 0 is the neutral element of the group $(Q,+)$ and $\varepsilon: Q \to Q, \varepsilon(x)=x, \forall x \in Q$.*

Proof. So as $((Q,+),\varphi,\psi,g)$ is a T-form of $(Q,\cdot)$ we have $x\cdot y = \varphi(x)+\psi(y)+g, \forall x,y \in Q$, so the identity (2) implies:

$$\varphi(x)+\psi(\varphi(y)+\psi(\varphi(y)+\psi(x)+g)+g)+g = y \Leftrightarrow$$

$$\varphi(x)+\psi\varphi(y)+\psi^2\varphi(y)+\psi^3(x)+\psi^2(g)+\psi(g)+g = y, \tag{17}$$

$\forall x,y \in Q$. Taking $x=y=0$ in (17), we have $\psi^2(g)+\psi(g)+g=0$, so (17) is equivalent to

$$\varphi(x)+\psi\varphi(y)+\psi^2\varphi(y)+\psi^3(x) = y, \tag{18}$$

$\forall x,y \in Q$. Now, taking $y=0$ and, after this $x=0$, in (18) we get $\varphi+\psi^3=\omega \Leftrightarrow \varphi = I\psi^3$, and respectively, $\psi\varphi+\psi^2\varphi=\varepsilon$ or $\psi^2+\psi=\varphi^{-1}=\psi^{-3}I \Leftrightarrow \psi^6+\psi^4=I$, where $\omega: Q \to Q, \omega(x)=0, \ \forall x \in Q$.

Conversely, if the conditions 1), 2) and 3) hold, then $y=\varepsilon(y)+\omega(x)=\psi\varphi(y)+\psi^2\varphi(y)+\varphi(x)+\psi^3(x)+\psi^2(g)+\psi(g)+g = x\cdot(y\cdot yx)$, so $(Q,\cdot)$ is a π-quasigroup of type T_2. □

Corollary. *π-T-Quasigroups of type T_2 are medial quasigroups.*

Proof. If $(Q,\cdot)$ is a π-T-quasigroup of type T_2 then $\varphi = -\psi^3$, so $\varphi\psi = -\psi^4 = \psi(-\psi^3) = \psi\varphi$, i.e. $(Q,\cdot)$ is a medial quasigroup. □

Let $(Q,\cdot)$ be a quasigroup and let $Aut(Q,\cdot)$ be its group of automorphisms. Define on $H = Aut(Q,\cdot) \times Q$ the operation "$\circ$" as follows:

$$(\alpha,x)\circ(\beta,y) = (\alpha\beta,\beta(x)\cdot y),$$

$\forall (\alpha,x),(\beta,y) \in H$. Then $(H,\circ)$ is a quasigroup and is called the holomorph of $(Q,\cdot)$.

Proposition 7. *Let $(Q,\cdot)$ be a π-quasigroup of type T_2. Then its holomorph $(H,\circ)$ is a π-quasigroup of type T_2 if and only if $Aut(Q,\cdot)=\{\varepsilon\}$, where ε is the identical mapping on Q.*

Proof. The holomorph $(H,\circ)$ of the quasigroup $(Q,\cdot)$ is a π-quasigroup of type T_2 if and only if it satisfies the identity:

$$(\alpha,x)\circ[(\beta,y)\circ((\beta,y)\circ(\alpha,x))] = (\beta,y). \tag{19}$$

Using the definition of "$\circ$" in (19), we have:

$$(\alpha\beta^2\alpha,\beta^2\alpha(x)\cdot[\beta\alpha(y)\cdot(\alpha(y)\cdot x)]) = (\beta,y),$$

which imply, in particular, the equality $\alpha\beta^2\alpha = \beta, \forall\alpha,\beta \in Aut(Q,\cdot)$. Taking in the last equality $\alpha = \varepsilon$, we obtain $\beta = \varepsilon, \forall\beta \in Aut(Q,\cdot)$, so $Aut(Q,\cdot) = \{\varepsilon\}$. Conversely, if $Aut(Q,\cdot) = \{\varepsilon\}$, then $(H,\circ) \cong (Q,\cdot)$, so $(H,\circ)$ is a π-quasigroup of type T_2. □

Proposition 8. *If (Q,A) is a finite π-quasigroup of type T_1, then $|Q| \equiv 0$ or $1 \ (mod\ 3)$.*

Proof. Let (Q,A) be a finite π-quasigroup of type T_1 and let $|Q| = q$. The quasigroup (Q,A) satisfies the identity

$$A\left(x, A(x, A(x,y))\right) = y. \tag{20}$$

Denoting the binary selectors, defined on the set Q, by F and E, i.e.
$F(x,y) = x, E(x,y) = y, \forall x,y \in Q$, the equality (20) implies:
$E = A(F, A(F,A)) = A(F(F,A), A(F,A)) = A(F,A)^2 \Rightarrow E(F,A) = A(F,A)^3 \Rightarrow A(F,E) = A = A(F,A)^3 \Rightarrow (F,E) = (F,A)^3$
so

$$(F,A)^3 = \varepsilon_{Q^2}, \tag{21}$$

where ε_{Q^2} is the identical mapping on Q^2 and
$(F,A): Q^2 \to Q^2, (F,A)(x,y) = (F(x,y), A(x,y)) = (x, A(x,y))$.

Denoting $(F,A) = \alpha$ and using (21), we get

$$\alpha^3 = \varepsilon_{Q^2}. \tag{22}$$

So as (Q,A) is a quasigroup, $F \perp A$, so $\alpha = (F,A)$ is a bijection. Remark that, for $(i,j) \in Q^2$, we have $\alpha(i,j) = (i,j) \Leftrightarrow (F,A)(i,j) = (i,j) \Leftrightarrow (i, A(i,j)) = (i,j) \Leftrightarrow A(i,j) = j \Leftrightarrow i$ is the local left unit of j. Denoting the left local unit of j by f_j, we obtain the set U of all elements from Q^2, which are invariant under α: $U = \{(f_j, j) | j = 1, 2, \dots, q\}$. Hence, exactly q elements from Q^2 are invariant, under $\alpha = (F,A)$, i.e. the rest of $q^2 - q$ elements are not invariant. Now, using (22) we obtain that α is a product of cycles of length 3 on a set of $q^2 - q = q(q-1)$ elements, i.e. $q \equiv 0$ or $1 \ (mod\ 3)$. □

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